UNSTEADY FRICTIONAL HEATING OF PROJECTIONS OF MICROIRREGULARITIES OF A SLIDING CONTACT

V. A. Galazyuk, A. A. Evtushenko, and I. N. Turchin

UDC 536.12:621.891

A nonstationary mixed problem of heat conduction that models the process of frictional heating in sliding of rough surfaces is solved.

It is known that the interaction of rough surfaces within a minimal contact area can bring about highly localized rapid temperature bursts. Determination of the surface temperature of friction pairs experimentally is difficult, due to a number of factors [1, 2]. Therefore, such investigations have been carried out theoretically [3-5] assuming that the actual area of contact coincided with its nominal value. This made it possible to use the theory of a fast-moving source [6]. However, if the projections of microirregularities are sufficiently widely spaced (the distance between adjacent projections exceeds by an order of magnitude the size of an individual surface irregularity), then we can carry out an analysis for each irregularity separately assuming their mutual effect to be of no importance.

Let us consider an elastic half-space whose surface $z = 0$ is rough. The region near the apex of an individual microprojection is modeled by an element of a spherical segment of radius R. We assume that due to contact with the other half-space, an individual apex experiences normal and shear forces, P and fP, respectively. The bodies are in sliding contact: the surface of the half-space slides with a constant velocity V over a fixed spherical irregularity in the direction of a certain axis x. Friction on the contact area generates heat that forms a heat flux directed towards the inside of the fixed body:

$$Q(r) = \gamma f V p(r), \quad r = \sqrt{x^2 + y^2}, \quad r \le R.$$

Here $p(r)$ is the contact pressure in a corresponding isothermal problem [7]:

$$p(r) = P_0 \sqrt{1 - \rho^2}, \quad \rho = r/R,$$

On the remaining portion of the body we take into account convective heat transfer.

Thus, to investigate the thermal regime of a microprojection, we must solve the heat conduction equation:

$$\partial^2_{\rho\rho} T + \rho^{-1} \partial_\rho T + \partial^2_{zz} T = \partial_{\mathrm{Fo}} T; \tag{1}$$

subject to the initial

$$T(\rho, Z, 0) = 0 \tag{2}$$

and mixed boundary conditions

$$\partial_z T = \begin{cases} -\Lambda \sqrt{1 - \rho^2}, & Z = 0, \quad \rho < 1, \\ \mathrm{Bi}\, T, & Z = 0, \quad \rho > 1, \end{cases} \tag{3}$$

I. Franko L'vov State University, Ukraine. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 69, No. 5, pp. 768-772, September-October, 1996. Original article submitted August 16, 1994.

$$T \to 0 \quad \text{when} \quad \sqrt{\rho^2 + Z^2} \to \infty , \tag{4}$$

where $\Lambda = \gamma f V P_0 R / \lambda_1$.

An analytical solution of problem (1)-(4) at Bi = 0 was obtained in [5].

For values of Bi that differ from zero, we construct a solution of the initial boundary-value problem of heat conduction (1)-(4) by applying the Chebyshev–Laguerre integral transformation [8] with respect to the dimensionless parameter Fo:

$$T_n(\rho, Z) = \int_0^\infty e^{-\lambda \mathrm{Fo}}\, T(\rho, Z, \mathrm{Fo})\, L_n(\lambda \mathrm{Fo})\, d\mathrm{Fo}, \quad n = 0, 1, 2, \dots , \tag{5}$$

$$T(\rho, Z, \mathrm{Fo}) = \lambda \sum_{n=0}^{\infty} T_n(\rho, Z)\, L_n(\lambda \mathrm{Fo}) \tag{6}$$

and the Hankel integral transform with respect to the radial variable.

In Eqs. (5) and (6) λ is a parameter playing the part of a scale factor and serving to economize series (7) over different ranges of the variable Fo (selected from the condition that $0.1 < \lambda \mathrm{Fo} < 5$).

As a result, the heat conduction equation (1) is reduced to a sequence of ordinary differential equations of the form:

$$d^2_{zz}\overline{T}_n - (\lambda + \xi^2)\, \overline{T}_n = \lambda \sum_{m=0}^{n-1} \overline{T}_m , \quad n = 0, 1, 2, \dots , \tag{7}$$

where $\overline{T}_n(\xi, Z) = \int_0^\infty \rho T_n(\rho, Z) J_0(\xi\rho) d\rho$ is the Hankel transform.

We represent a general solution of the sequence of differential equations (7) as

$$\overline{T}_n(\xi, Z) = \sum_{j=0}^{n} [A_{n-j}(\xi)\, G_j(\xi, Z) + B_{n-i}(\xi)\, W_j(\xi, Z)] , \tag{8}$$

where $A_{n-j}(\xi)$ and $B_{n-j}(\xi)$ are functions to be determined from boundary conditions (3) and (4), whereas $G_j(\xi, Z)$ and $W_j(\xi, Z)$ are linearly independent fundamental systems of the solution of sequence (7). Since $W_j(\xi, Z) \to \infty$ when $Z \to \infty$, then condition (4) immediately yields $B_k(\xi) \equiv 0$, $k = 0, 1, 2, \dots$. Using the method of undetermined coefficients, we represent the functions $G_j(\xi, Z)$ as

$$G_j(\xi, Z) = \exp\left(-Z\sqrt{\xi^2 + \lambda}\right) \sum_{k=0}^{j} a_{j,k}(\xi)\, Z^k . \tag{9}$$

Substituting Eq. (9) into sequence (7), we obtain recurrent relations for determining the unknowns $a_{j,k}(\xi)$:

$$a_{j,p+1} = 0.5\,(\xi^2 + \lambda)^{-1/2} (p+1)^{-1} \left\{ (p+1)(p+2)\, a_{j,p+2} + \lambda \sum_{k=p}^{j-1} a_{k,p} \right\} ,$$
$$j = 1, 2, 3, \dots ; \quad k = 0, 1, \dots , j-1 . \tag{10}$$

We note that in relations (10) $a_{j,k}(\xi) \equiv 0$, when $k > j$, and $a_{j,0}(\xi)$ are arbitrary functions. Suppose $a_{j,0}(\xi) = \delta_{j0}$; then it follows from Eq. (9) that $G_j(\xi, 0) = \delta_{j0}$ too.

Having satisfied boundary condition (3), transformed according to formula (5), we arrive at a sequence of paired integral equations of the form

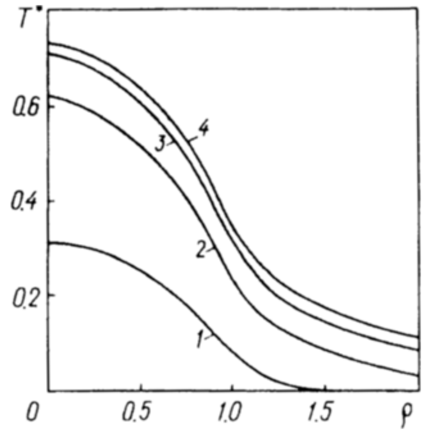

Fig. 1. Distribution of the dimensionless temperature T^* on the half-space surface ($Z = 0$) at Bi = 0.1: 1) Fo = 0.1, 2) 1.1, 3) 6.4, 4) Fo = ∞.

$$\int_0^\infty \xi \sum_{j=0}^{n} A_{n-j}(\xi)\, G_j'(\xi)\, J_0(\rho\xi)\, d\xi = -\frac{\Lambda}{\lambda}\sqrt{1-\rho^2}\,\delta_{0n}, \tag{11}$$

$$\rho < 1, \quad n = 0, 1, 2, \dots;$$

$$\int_0^\infty \xi \left[\sum_{j=0}^{n} A_{n-j}(\xi)\, G_j'(\xi) - \mathrm{Bi}\, A_n(\xi)\right] J_0(\rho\xi)\, d\xi = 0, \tag{12}$$

$$\rho > 1, \quad n = 0, 1, 2, \dots,$$

where $G_j'(\xi) \equiv \partial_z G_j(\xi, Z)|_{Z=0}$.

Since $G_0'(\xi) = -\sqrt{\xi^2 + \lambda}$, we write the solution of sequence (11), (12) using a Neumann series as

$$A_n(\xi) = \frac{1}{\sqrt{\xi^2+\lambda} + \mathrm{Bi}}\left[\frac{1}{\xi}\sum_{k=0}^{\infty} a_k^n J_{2k+1}(\xi) + \sum_{j=0}^{n} A_{n-j}(\xi)\, G_j'(\xi)\right]. \tag{13}$$

On the basis of the well-known [9] properties of the Weber–Schafheitlin discontinuous integral Eq. (12) is satisfied identically, and Eq. (11), after some transformations [10], yields a sequence of infinite systems of linear algebraic equations

$$\tilde{a}_m^n + \sum_{k=0}^{\infty} \tilde{a}_k^n b_{km} = c_m^n, \quad m = 0, 1, 2, \dots; \quad n = 0, 1, 2, \dots, \tag{14}$$

where

$$b_{km} = 2(2k+1)^{1/2}(2m+1)^{1/2}\int_0^\infty \frac{J_{2k+1}(\xi)\, J_{2m+1}(\xi)}{\xi(\sqrt{\xi^2+\lambda} + \mathrm{Bi})}\, d\xi;$$

$$c_m^n = 2(2m+1)^{1/2}\left[\Lambda\delta_{0n}\int_0^\infty \xi^{-3/2} J_{3/2}(\xi)\, J_{2m+1}(\xi)\, d\xi + \right.$$

$$\left. + \mathrm{Bi}\sum_{j=1}^{n}\int_0^\infty \frac{A_{n-j}(\xi)\, G_j'(\xi)\, J_{2m+1}(\xi)}{\sqrt{\xi^2+\lambda} + \mathrm{Bi}}\, d\xi;\right] \quad \tilde{a}_m^n = (2m+1)^{-1/2} a_m^n.$$

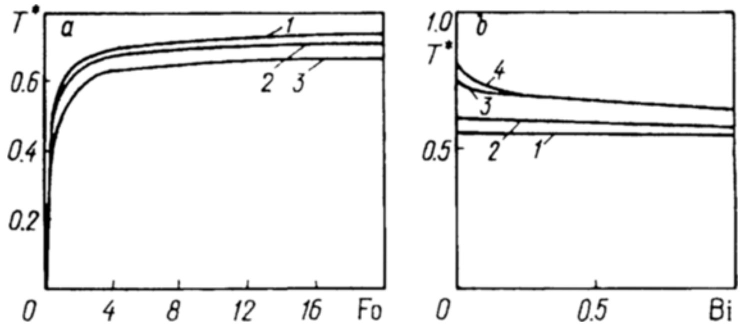

Fig. 2. Dependence of the dimensionless temperature T^* at the center of the heating spot ($Z = 0$; $\rho = 0$) on the dimensionless numbers Fo (1) Bi = 0, 2) 0.1, 3) 1) (a) and Bi (1) Fo = 0.1, 2) 1.2, 3) 10.1, 4) Fo = ∞) (b).

We can show that

$$\sum_{k,m=0}^{\infty} (b_{km})^2 < \infty , \quad \sum_{m=0}^{\infty} (c_m^n)^2 < \infty , \quad n = 0, 1, 2, \ldots ,$$

and, according to [11], a solution of systems (14) that satisfies the condition

$$\sum_{k=0}^{\infty} (a_k^n)^2 < \infty , \quad n = 0, 1, 2, \ldots ,$$

exists and can be found by, say, the method of reduction.

Having determined a_k^n from the sequence of algebraic equations (14) and, correspondingly, $\Lambda_n(\xi)$ from relation (13), we find a solution of the initial boundary value problem (1)-(4) in the form

$$T(\rho, Z, \mathrm{Fo}) = \lambda \sum_{n=0}^{\infty} L_n(\lambda \mathrm{Fo}) \sum_{j=0}^{n} \int_0^{\infty} \xi A_{n-j}(\xi)\, G_j(\xi, Z)\, J_0(\xi\rho)\, d\xi . \tag{15}$$

Numerical Analysis. The dependence of the dimensionless temperature $T^* = \Lambda^{-1}T$ on the radial variable for four different values of the Fourier number at $Z = 0$, Bi = 0.1 is presented in Fig. 1. We judge the attainment of a steady state near the source by an insignificant change in temperature when the time variable is increased by an order of magnitude. At points closer to the heat source, the transition process is shorter. For example, at Fo = 1.1 the temperature at the center accounts for 43.5% of the stationary value, while at point $\rho = 2.0$, for only 7%. A sharp temperature gradient is established near the heat source (for example, at values of Fo of the order of unity), while in the region with $\rho > 2$, the temperature is preserved almost at a zero level.

Transition processes of the change of temperature at the center of the region of heating for Bi = 0, 0.1, 1 are presented in Fig. 2a. For analysis of the orders of magnitude, we assume that a steady state near the source corresponds to Fo ≈ 10. For the majority of metals $k \approx 10^{-5}$ m^2/sec, so that at $R = 1$ and 100 μm the characteristic times of the transition process are 0.01 and 100 msec, respectively.

Figure 2b presents the nonstationary temperature of the center of the heating spot versus Bi for four different values of Fo. The maximum value of the dimensionless temperature $T^*_{\max} = \pi/4 = 0.78$ is attained for $Fo \to \infty$ and Bi → 0, so that $T_{\max} = \Lambda\pi/4 = (\gamma f V P_0 R/\lambda_t)(\pi/4)$. Thus, with an increase in the source radius under fixed frictional conditions on the contact, the maximum temperature increases. Since the space coordinates are related to the heating spot radius R, the results presented here are valid for any actual size of the contact region that satisfies the requirement of its smallness compared to the body dimensions. In particular, it was established that a contact spot causes a temporary localized increase in the surface temperature within the limits of $\rho \le 2$. The results obtained in the present work can be related to the problem of a moving body with a microprojection on the surface when the duration of contact exceeds the characteristic time of the temperature transition process.

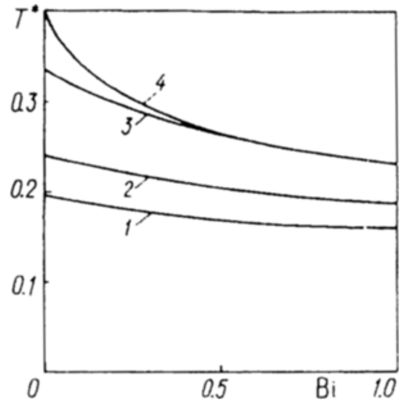

Fig. 3. The dimensionless temperature T^* at the separation line of boundary conditions ($Z = 0$; $\rho = 1$) vs the dimensionless number Bi at 1) Fo = 0.1, 2) 1.1, 3) 10.1, 4) Fo = ∞.

A different picture is observed in Fig. 3, which presents the same results for the point $\rho = 1$ (the line of separation of boundary conditions). We can see a steep rise in the temperature profile in the steady state when Bi $\to 0$. This implies that in the region with $\rho \simeq 1$ convective heat transfer on the surface is an significant factor.

Conclusions. It is found that the influence of a local spot of contact is localized within the limits of the region $\rho \leq 2$, and that a substantial increase in temperature occurs only within the region of $\rho \leq 1$. The temperature field turned out to be highly localized with sharp gradients in both the axial and radial directions.

It is shown that convective cooling of the surface influences both the level of the local increase in temperature and the extent of the heating region in the radial and axial directions.

The characteristic durations of transition processes in heating that were recorded in the experiments of [2], which are on the order of 0.1–1 msec, agree with the theoretical estimates of the present work.

NOTATION

T, temperature of the half-space; R, radius of the surface microirregularity; $\rho = r/R$, $Z = z/R$, dimensionless variables in a cylindrical coordinate system; $\mathrm{Fo} = kt/R^2$, Fourier number; $\mathrm{Bi} = hR/\lambda_t$, Biot number; $P_0 = 3/2(P/\pi R^2)$, maximum Hertz pressure; P, load on contact; λ_t, thermal conductivity coefficient; h, heat transfer coefficient; k, thermal diffusivity coefficient; f, friction factor; V, sliding velocity; γ, coefficient of separation of heat fluxes; $L_n(x)$, Chebyshev–Laguerre polynomials; $J_\nu(x)$, Bessel function; δ_{j0}, the Kronecker delta.

REFERENCES

1. A. V. Chichinadze, R. M. Matveyevskii, É. P. Braum, et al., Materials in the Tribotechnology of Unsteady Processes [in Russian], Moscow (1986).
2. N. N. Bogdanovich and V. N. Belov, Trenie Iznos, 13, No. 4, 624-632 (1992).
3. H. Blok, SAE Trans, 64, 54-63 (1940).
4. F. P. Bouden and D. Teibor, "The surface temperature of rubbing bodies," in: Friction and Boundary Lubrication (ed. I. V. Kragel'skii) [in Russian], pp. 231-254.
5. A. A. Yevtushenko and O. M. Ukhanskaya, J. Appl. Math. Mech., 56, No. 1, 95-101 (1992).
6. J. C. Jaeger, J. and Proc. Roy. Soc. N. S. W. Walles, 56, Pt. 3, 203-224 (1942).
7. K. Jonson, Mechanics of Contact Interaction [Russian translation], Moscow (1989).
8. V. A. Galazyuk and I. M. Turchin, Dop. Akad. Nauk Ukraini, No. 1, 23-27 (1993).
9. G. N. Watson, Theory of Bessel Functions [Russian translation], Pt. 1, Moscow (1949).
10. C. J. Tranter, Quart. Journ. Mech. Appl. Math., VII, Pt. 3, 317-325 (1954).
11. L. V. Kantorovich and G. P. Akilov, Functional Analysis [in Russian], Moscow (1977).